Mathematics

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The Undergraduate Program

The mathematics department offers a wide range of courses in pure and applied mathematics for its majors and for students in other disciplines. The department offers six majors leading to the B.A. degree: mathematics, applied mathematics, mathematics–applied science, mathematics–computer science, joint major in mathematics and economics, and mathematics–secondary education, and one leading to the B.S. degree: mathematics–scientific computation. In addition, students can minor in mathematics. The department also has an Honors Program for exceptional students in any of the seven majors. See the sections on major programs and the other areas mentioned above as well as the course descriptions at the end of this section for more specific information about program requirements and the courses that are offered by the department. You may visit our Web site, math.ucsd.edu for more information including course Web pages, career advising, and research interests of our faculty.

First-Year Courses

Entering students must take the Mathematics Placement Exam (MPE) prior to registration unless they have an appropriate score on an AP calculus exam, a score of 650 or higher on the SAT II Math Level 2C Exam, an appropriate score on the International Baccalaureate Higher Level Mathematics Exam, credit by means of a foreign exam (e.g., GCE), or transferable college credit in calculus. The purpose of the MPE is to recommend placement for entering students in Math. 3C, 4C, 10A, or 20A. For more information about the MPE (test dates, test description, sample exams, online practice tests), see the Mathematics Testing and Placement Web site, http://mathtesting.ucsd.edu.

Prerequisites for Math. 4C, 10A-B-C and 20A-B-C-D-E-F are enforced through TritonLink. Students need to ensure that test scores and transferable college credit are submitted to the Registrar prior to enrollment through WebReg.

Math. 3C is the department’s preparatory course for the Math. 10 sequence, providing a review of algebraic skills, facility in graphing, and working with exponential and logarithmic functions. Math. 4C is the department’s preparatory course for the Math. 20 sequence, providing a brief review of college algebra followed by an introduction to trigonometry and a more advanced treatment of graphing and functions. Math. 10A-B-C is one of two calculus sequences. The students in this sequence have completed a minimum of two years of high school mathematics. This sequence is intended for majors in liberal arts and the social and life sciences. It fulfills the mathematics requirements of Revelle College and the option of the general-education requirements of Muir College. Completion of two quarters fulfills the requirement of Marshall College and the option of Warren College and Eleanor Roosevelt College.
As of summer 2003, Math. 21C and 21D have requirements. Any mathematics course numbered be counted toward fulfillment of the major receive a grade of C– or better in any course to in the upper-division courses used to satisfy the required for some science and engineering majors. provides the foundation for Math. 20D-E-F which is strongly encouraged to take Math. 20 since it provides adequate backgrounds in mathematics are strongly encouraged to take Math. 20 since it provides the foundation for Math. 20D-E-F which is required for some science and engineering majors. Note: As of summer 2003, Math. 21C and 21D have been renumbered to Math. 20C and 20D.

Certain transfers between the Math. 10 and Math. 20 sequences are possible, but such transfers should be carefully discussed with an adviser. Able students who begin the Math. 10 sequence and who wish to transfer to the Math. 20 sequence, may follow one of three paths:

1. Follow Math. 10A with Math. 20A, with two units of credit given for Math. 20A. This option is not available if the student has credit for Math. 10B or Math. 10C.
2. Follow Math. 10B with Math. 20B, receiving two units of credit for Math. 20B.
3. Follow Math. 10C with Math. 20B, receiving two units of credit for Math. 20B and two units of credit for Math. 20C.

Credit will not be given for courses taken simultaneously from the Math. 10 and the Math. 20 sequence.

Major Programs

The department offers six different majors leading to the Bachelor of Arts degree: (1) mathematics, (2) applied mathematics, (3) mathematics–applied science, (4) mathematics–computer science, (5) joint major in mathematics and economics, (6) mathematics–secondary education; and one leading to a B.S. degree: mathematics–scientific computation. The specific emphases and course requirements for these majors are described in the following sections. All majors must obtain a minimum 2.0 grade-point average in the upper-division courses used to satisfy the major requirements. Further, the student must receive a grade of C– or better in any course to be counted toward fulfillment of the major requirements. Any mathematics course numbered 100–194 may be used as an upper-division elective. (Note: 195, 196, 197, 198, 199, and 199H cannot be used towards any mathematics major.) All courses used to fulfill the major must be taken for a letter grade. No more than three upper-division courses taken externally from UCSD can be counted towards any major. Special exceptions may be considered via petition.

It is strongly recommended that all mathematics majors review their programs at least annually with a departmental adviser, and that they consult with the Advising Office in AP&M 6016 before making any changes to their programs. Current course offering information for the entire academic year is maintained on the department’s Web page at http://math.ucsd.edu. Special announcements are also emailed to all majors.

Students who plan to go on to graduate school in mathematics should be advised that only the best and most motivated students are admitted. Many graduate schools expect that students will have completed a full sequence of abstract algebra (Math. 100A-B-C) as well as a full sequence of analysis (Math. 140A-B-C). The advanced Graduate Record Exam (GRE) often has questions that pertain to material covered in the last quarter of analysis or algebra. In addition, it is advisable that students consider Summer Research Experiences for Undergraduates. This is a program funded by the National Science Foundation to introduce students to math research while they are still undergraduates. In their senior year or earlier, students should consider taking some graduate courses so that they are exposed to material taught at a higher level. In their junior year, students should begin to think of obtaining letters of recommendation from professors who are familiar with their abilities.

Math. 20D, E, and F do not need to be taken in order. Math majors are strongly advised to take 20F as early as possible after successfully completing 20C.

Major in Mathematics

The upper-division curriculum provides courses for students who will use mathematics as a tool in the biological, physical and behavioral sciences, and the humanities.

REQUIRED COURSES:

Lower-Division

1. Calculus: Math. 20A-B-C-D-E-F

Upper-Division


3. One of the following sequences:
   b. Advanced Calculus: Math. 142A-B and Math. 120A (Math. 140A-B-C recommended for graduate school)

4. One of the following sequences:
   a. Modern Algebra: Math. 100A-B
   b. Modern Applied Algebra: Math. 103A-B and Math. 102 (Math. 100A-B-C recommended for graduate school)

5. Upper-division electives to complete thirteen four-unit courses, chosen from any mathematics course numbered between 100 and 194 (including those taken from the requirements listed above.)

As with all departmental requirements, more advanced courses on the same material may be substituted with written approval from the departmental adviser. To be prepared for a strong major curriculum, students should complete the last three quarters of the 20 sequence and Math. 109 before the end of their sophomore year. Either Math. 140A-B or 100A-B should be taken during the junior year.

Major in Applied Mathematics

A major in applied mathematics is also offered. The program is intended for students planning to work on the interface between mathematics and other fields.
Major in Mathematics–
Scientific Computation

This major is designed for students with a substantial interest in scientific computation. The program is a specialized applied mathematics program with a concentration in computer solutions of scientific problems.

REQUIRED COURSES:

Lower-Division

1. Calculus: Math. 20A-B-C-D-E-F
2. Programming (one of the following)
   a. CSE 8A-B (Intro to Computer Sci: Java)
   b. CSE 11 (Intro to Computer Sci: Java, Accelerated Pace)
   c. MAE 9 (C/C++ Programming)
   d. MAE 10 (FORTRAN for Engineers)

Upper-Division

4. Linear Algebra: Math. 102 or Math. 170A
5. Advanced Calculus: Math. 140A-B or Math. 142A-B
6. One of the following sequences:
   a. Math. 180A-B-C-181A
   b. Math. 180A-181A and any two from Math. 181B-C-D-E
   c. (Math. 183 or Math. 180A-181A) and any three from Math. 170A-B-C-172-173
7. One additional sequence which may be chosen from the list (#6) above or the following list: Math. 110-120A-130A, 120A-B, 130A-132A, 155A-B, 171A-B, 193A-B
8. Upper-division electives to complete at least thirteen four-unit courses, chosen from any mathematics course numbered between 100 and 194 (including those taken from the requirements listed above) except:
   a. Up to twelve units may be taken from outside the department in an approved applied mathematical area. A petition specifying the courses to be used must be approved by an applied mathematics adviser. No such units may also be used for a minor or program of concentration.
   b. MAE 107, Econ. 120A-B-C, cannot be counted toward the thirteen required courses.

To be prepared for a strong major curriculum, students should complete the last three quarters of the 20 sequence and Math. 109 before the end of their sophomore year.

Major in Mathematics–
Applied Science

This major is designed for students with a substantial interest in mathematics and its applications to a particular field such as physics, biology, chemistry, biochemistry, cognitive science, computer science, economics, management science, or engineering.

REQUIRED COURSES:

Lower-Division

1. Calculus: Math. 20A-B-C-D-E-F
2. Programming (one of the following is recommended):
   a. CSE 8A-B (Intro to Computer Sci: Java)
   b. CSE 11 (Intro to Computer Sci: Java, Accelerated Pace)
   c. MAE 9 (C/C++ Programming)
   d. MAE 10 (FORTRAN for Engineers)

Upper-Division Mathematics Requirements:

4. Linear Algebra: Math. 102 or Math. 170A
5. Advanced Calculus: Math. 140A-B or Math. 142A-B
6. One of the following sequences:
   a. Math. 180A-B-C-181A
   b. Math. 180A-181A and any two from Math. 181B-C-D-E
   c. (Math. 183 or Math. 180A-181A) and any three from Math. 170A-B-C-172-173
7. One additional sequence which may be chosen from the list (#6) above or the following list: Math. 110-120A-130A, 120A-B, 130A-132A, 155A-B, 171A-B, 193A-B
8. Upper-division electives to complete at least thirteen four-unit courses, chosen from any mathematics course numbered between 100 and 194 (including those taken from the requirements listed above) except:
   a. Up to 3 upper-division courses may be taken outside the department in an approved applied mathematical area. A petition specifying the courses to be used must be approved by an applied mathematics adviser. No such units may also be used for a minor or program of concentration.
   b. MAE 107, Econ. 120A-B-C, cannot be counted toward the thirteen required courses.

Upper-Division Applied Science Requirements:

7. Seven upper-division courses selected from one or two other departments (these cannot be from mathematics). At least three of these seven upper-division courses must require at least Math. 20C as a prerequisite.

Students must submit an individual plan for approval in advance by a mathematics department adviser, and all subsequent changes in the plan must be approved by a mathematics department adviser.

Major in Mathematics–
Computer Science

The program provides for a major in computer science within the Department of Mathematics. Graduates of this program will be
mathematically oriented computer scientists who have specialized in the mathematical aspects and foundations of computer science or in the computer applications of mathematics.

As of fall 2000, a mathematics-computer science major is not allowed to also minor in computer science in the Computer Science and Engineering department.

The detailed curriculum is given in the list below:

**Mathematics–Computer Science Pre-Major**

In October 2001, the Academic Senate approved a minimum GPA requirement of 2.5 in the lower-division mathematics courses required for the mathematics–computer science major. The 2.5 minimum GPA in the lower-division math courses reflects minimal preparation for the upper-division courses required for the major.

Therefore, students entering UCSD as first-year students for the fall 2002 quarter and later and students entering as transfer students for the fall 2003 quarter and later will be held to this requirement. Applications from students entering UCSD on or after the effective dates above will be held until all lower-division math courses for the major are completed and the minimum GPA in those courses of 2.5 can be verified. Students meeting the 2.5 minimum GPA requirement will be accepted into the mathematics–computer science major.

**REQUIRED COURSES:**

**Lower-Division**

1. Calculus: Math. 20A-B-C-D-F
2. Intro to Computer Science—CSE 8A-B
   Introduction to Computer Science: Java, or CSE 11 Introduction to Computer Science: Java (Accelerated)
3. Basic Data Structures and Object-oriented Programming: CSE 12
4. Computer Organization and Systems Programming: CSE 30 (Note: CSE 30 requires CSE 20 or Math. 15A as a prerequisite)

**Upper-Division**

7. Theory of Computability: Math. 166 (or CSE 105)
8. Intro to Probability: Math. 180A or 183
9. Mathematical Foundations of Computer Science: Math. 184A
10. Computer Implementations of Data Structures: Math. 176 (or CSE 100)
11. Design & Analysis of Algorithms: Math. 188 (or CSE 101)

In order to graduate by the end of their senior year, students should complete Math. 103A-B by the end of their junior year.

**Joint Major in Mathematics and Economics**

Majors in mathematics and the natural sciences often feel the need for a more formal introduction to issues involving business applications of science and mathematics. Extending their studies into economics provides this application and can provide a bridge to successful careers or advanced study. Majors in economics generally recognize the importance of mathematics to their discipline. Undergraduate students who plan to pursue doctoral study in economics or business need the more advanced mathematics training prescribed in this major.

This major is considered to be excellent preparation for Ph.D. study in economics and business administration, as well as for graduate studies for professional management degrees, including the MBA. The major provides a formal framework making it easier to combine study in the two fields.

Course requirements of the Joint Major in Mathematics and Economics consist principally of the required courses of the pure mathematics major and the economics/management science major.

**REQUIRED COURSES:**

**Lower-Division**

1. Calculus: Math. 20A-B-C-D-F
2. Intro to Economics: Econ. 1-2-3

**Upper-Division**

Fifteen upper-division courses in mathematics and economics, with a minimum of seven courses in each department, chosen from the courses listed below (prerequisites are strictly enforced):

4. One of the following:
   Applied Linear Algebra: Math. 102
   Numerical Linear Algebra: Math. 170A
   Modern Algebra: Math. 100A-B
5. One of the following:
   Foundations of Analysis: Math. 140A
   Advanced Calculus: Math. 142A
6. One of the following:
   Ordinary Differential Equations: Math. 130A
   Foundations of Analysis: Math. 140B
   Advanced Calculus: Math. 142B
7. One of the following:
   Microeconomics: Econ. 100A-B or Management Science Microeconomics: Econ. 170A-B
8. Econometrics: Econ. 120A-B-C or Math. 180A and Econ. 120B-C or Probability: Math. 180A, 181A and Econ. 120C
9. One of the following:
   Macroeconomics: Econ. 110A-B
   Mathematical Programming: Numerical Optimization: Math. 171A-B
   or
   Two courses from the following:
   Decisions Under Uncertainty: Econ. 171 Introduction to Operations Research: Econ. 172A-B-C, (Note: 172A is a prerequisite for 172BC)
   Other courses which are strongly recommended are: Math. 130B, 131, 181B, 193A-B and 194 and Econ. 109, 113, 175, and 178.

**Major in Mathematics–Secondary Education**

This major offers excellent preparation for teaching mathematics in secondary schools. Students interested in earning a California teaching credential from UCSD should contact the
Teacher Education Program (TEP) for information regarding prerequisites and requirements. It is recommended you contact TEP as early as possible.

**Lower-Division Requirements**

1. Calculus 20A-B-C-D-E-F
   
   reopen recommended you contact TEP as early as possible.
   
   **Upper-Division Requirements:**
   
   4. Number Theory: Math. 104A
   5. History of Mathematics: Math. 163
   6. Practicum in Learning: TEP 129A-B-C
   7. One of the following:
      - Computer Algebra: Math. 107A
      - Computer Graphics: Math. 155A
      - Mathematical Computing: Math. 161
      - Numerical Linear Algebra: Math. 170A
      - Intro. to Cryptography: Math. 187
   8. One of the following:
      - Intro. to Probability: Math. 180A
      - Statistical Methods: Math. 183
   9. One of the following:
      - Differential Geometry: Math. 150A
      - Topics in Geometry: Math. 151
      - Geometry for Secondary Teachers: Math. 153
      - Intro. to Topology: Math. 190
   10. One of the Following:
        - Modern Algebra: Math. 100A
        - Applied Linear Algebra: Math. 102
        - Modern Applied Algebra: Math. 103A
   11. One of the following:
        - Foundations of Analysis: Math. 140A
        - Advanced Calculus: Math. 142A
   12. Upper-division courses must total thirteen courses. Upper-division courses must include at least one two-quarter sequence from the following list:

**Minor in Mathematics**

The minor in mathematics consists of seven or more courses. At least four of these courses must be upper-division courses taken from the UCSD Department of Mathematics. Acceptable upper-division courses are Math. 20D, 20E, and 20F.

Math. 195, 196, 197, 198, 199, and 199H are not acceptable courses for the mathematics minor. A grade of C– or better (or P if the Pass/No Pass option is used) is required for all courses used to satisfy the requirements for a minor. There is no restriction on the number of classes taken with the P/NP option. Upper-division courses cannot overlap between major and minor programs.

**Mathematics Honors Program**

The Department of Mathematics offers an honors program for those students who have demonstrated excellence in the major. Successful completion of the honors program entitles the student to graduate with departmental honors (see Department Honors in the Academic Regulations section). Application to the program should be made the spring quarter before the student is at senior standing.

**Requirements for admission to the program are:**

1. Junior standing
2. An overall GPA of 3.0 or higher
3. A GPA in the major of 3.5 or higher
4. Completion of Math. 109 (Mathematical Reasoning) and at least one of Math. 100A, 103A, 140A, or 142A. (Completion of additional major courses is strongly recommended.)

**Completion of the honors program requires the following:**

1. At least one quarter of the student colloquium, Math. 196 (Note: Math. 196 is only offered in the fall quarter.)
2. The minimum 3.5 GPA in the major must be maintained
3. An Honors Thesis. The research and writing of the thesis will be conducted over at least two quarters of the junior/senior years under the supervision of a faculty adviser. This research will be credited as eight to twelve units of Math. 199H. The completed thesis must be approved by the department’s Honors Committee, and presented orally at the Undergraduate Research Conference or another appropriate occasion.

The department’s Honors Committee will determine the level of honors to be awarded, based on the student’s GPA in the major and the quality of the honors work. Applications for the mathematics department’s Honors Program can be obtained at the mathematics department Undergraduate Affairs Office (AP&M 7018) or the Mathematics Advising Office (AP&M 6016). Completed applications can be returned to the Mathematics Advising Office.

**Duplication of Credit**

In the circumstances listed below, a student will not receive full credit for a Department of Mathematics course. The notation "Math. 20A [2 if Math. 10A previously/0 if Math. 10A concurrently/0 if Math. 10B or 10C]" means that a student already having credit for Math. 10A will receive only two units of credit for Math. 20A, but will receive no units if he or she has credit for Math. 10B or 10C, and no credit will be awarded for Math. 20A if Math. 10A is being taken concurrently.

1. Math. 4C [2 if Math. 10A previously/0 if Math. 20A previously or concurrently]
2. Math. 10A [0 if Math. 20A previously or concurrently]
3. Math. 10B [0 if Math. 20B previously or concurrently]
4. Math. 10C [0 if Math. 20C previously or concurrently]
5. Math. 15A [0 if CSE20 previously or concurrently]
6. Math. 15B [0 if CSE21 previously or concurrently]
7. Math. 20A [2 if Math. 10A previously/0 if Math. 10A concurrently/0 if Math. 10B or 10C]
8. Math. 20B [2 if Math. 10B or 10C previously/0 if Math. 10B concurrently]
9. Math. 20C [2 if Math. 10C previously/0 if Math. 10C concurrently]
10. Math. 20D [0 if Math. 10D previously]
11. Math. 20E [0 if Math. 2F previously]
12. Math. 20F [0 if Math. 2E previously]
13. Both Math. 100 and Math. 103 cannot be taken for credit
14. Math. 140A-B and Math. 142 A-B cannot both
be taken for credit
15. Math. 155A [0 if CSE 167]
16. Math. 166 [0 if CSE 105]
17. Math. 174 [0 if 170A or B or C previously]
18. Math. 176 [0 of CSE 100 previously or concurrently]
19. Math. 180A [2 if Econ. 120A previously/0 if Econ. 120A concurrently]
20. Math. 181A [2 if Econ. 120B/0 if Econ. 120B concurrently]
21. Math. 183 [0 if Econ. 120A or Math. 180A or Math. 181A has been taken previously or concurrently. Full credit for Math. 183 will be given if taken previously to Math. 180A or Math. 181A.]

Advisers
Advisers change yearly. Contact the undergraduate office at (858) 534-3590 for current information.

The Graduate Program
The Department of Mathematics offers graduate programs leading to the M.A. (pure or applied mathematics), M.S. (statistics), and Ph.D. degrees.

The application deadline for fall admission is January 15. Candidates should have a bachelor’s or master’s degree in mathematics or a related field from an accredited institution of higher education or the equivalent. A minimum scholastic average of B or better is required for course work completed in upper-division or prior graduate study. In addition, the department requires all applicants to submit scores no older than twelve months from both the GRE General Test and Advanced Subject Test in Mathematics. Completed files are judged on the candidate’s mathematical background, qualifications, and goals.

Departmental support is typically in the form of teaching assistantships, research assistantships, and fellowships. These are currently only awarded to students in the Ph.D. program.

General Requirements
All student course programs must be approved by a faculty adviser prior to registering for classes each quarter, as well as any changes throughout the quarter.

Full-time students are required to register for a minimum of twelve (12) units every quarter, eight (8) of which must be graduate-level mathematics courses taken for a letter grade only. The remaining four (4) units can be approved upper-division or graduate-level courses in mathematics-related subjects (Math. 500 may not be used to satisfy any part of this requirement). After advancing to candidacy, Ph.D. candidates may take all course work on a Satisfactory/Unsatisfactory basis. Typically, students should not enroll in Math. 299 until they have satisfactorily passed both qualifying examinations (see Ph.D. in Mathematics) or obtained approval of their faculty adviser.

Master of Arts in Pure Mathematics
[Offered only under the Comprehensive Examination Plan] The degree may be terminal or obtained on the way to the Ph.D. A total of forty-eight units of credit is required. Twenty-four of these units must be graduate-level mathematics courses approved in consultation with a faculty adviser.

In the selection of course work to fulfill the remaining twenty-four units, the following restrictions must be followed:

a. No more than eight units of upper-division mathematics courses.

b. No more than twelve units of graduate courses in a related field outside the department (approved by the Department of Mathematics).

c. No more than four units of Math. 295 (Special Topics) or Math. 500 (Apprentice Teaching).

d. No units of Math. 299 (Reading and Research) may be used in satisfying the requirements for the master’s degree.

COMPREHENSIVE EXAMINATIONS
Seven written departmental examinations are offered in three areas (refer to “Ph.D. in Mathematics,” Areas 1, 2, and 3, for list of exams). A student must complete two examinations, one from Area 1 and one from Area 2, both with an M.A. pass or better.

FOREIGN LANGUAGE REQUIREMENT
A reading knowledge of one foreign language (French, German, or Russian) is required. In exceptional cases other languages may be substituted. Testing is administered by faculty in the department who select published mathematical material in one of these languages for a student to translate.

TIME LIMITS
Full-time students are permitted seven quarters in which to complete all degree requirements. While there are no written time limits for part-time students, the department has the right to intervene and set individual deadlines if it becomes necessary.

Master of Arts in Applied Mathematics
[Offered only under the Comprehensive Examination Plan] The degree may be terminal or obtained on the way to the Ph.D. Out of the total forty-eight units of required credit, two applied mathematics sequences comprising twenty-four units must be chosen from the following list (not every course is offered each year):

261A-B, C. (Combinatorial Algorithms)
264A-B, C. (Combinatorics)
270A-B, C. (Numerical Mathematics)
271A-B, C. (Numerical Optimization)
273A-B, C. (Scientific Computation)

In certain cases, a petition may be approved to substitute one of these requirements from the following list of sequences:

220A-B, C. (Complex Analysis)
231A-B, C. (Partial Differential Equations)
240A-B, C. (Real Analysis)
280A-B, C. (Probability Theory)
281A-B, C. (Mathematical Statistics)
282A-B. (Applied Statistics)

In choosing course work to fulfill the remaining twenty-four units, the following restrictions must be followed:

a. At least eight units must be approved graduate courses in mathematics or other departments (a one-year sequence in a related area outside the department such as computer science, engineering, physics, or economics is strongly recommended);

b. A maximum of eight units can be approved upper-division courses in mathematics; and

c. A maximum of eight units can be approved upper-division courses in other departments.
d. A maximum of four units of Math. 500 (Apprentice Teaching).
e. NO UNITS of Math. 295 (Special Topics) or Math. 299 (Reading and Research) may be used.

Students are strongly encouraged to consult with a faculty adviser in their first quarter to prepare their course of study.

**COMPREHENSIVE EXAMINATIONS**

Two written comprehensive examinations must be passed at the master's level in any of the required applied mathematics sequences listed above. The instructors of each course should be contacted for exam details.

**FOREIGN LANGUAGE REQUIREMENT**

There is no foreign language requirement for the M.A. in applied mathematics.

**TIME LIMITS**

Full-time M.A. students are permitted seven quarters in which to complete all requirements. While there are no written time limits for part-time students, the department has the right to intervene and set individual deadlines if it becomes necessary.

**Master of Science in Statistics**

*(Offered only under the Comprehensive Examination Plan)* The M.S. in statistics is designed to provide recipients with a strong mathematical background and experience in statistical computing with various applications. Out of the forty-eight units of credit needed, required core courses comprise twenty-four units, including:

- Math. 281A-B. (Mathematical Statistics)
- Math. 282A-B. Applied Statistics

and any two topics comprising eight units chosen at will from Math. 287A-B-C-D and 289A-B-C (see course descriptions for topics).

The following guidelines should be followed when selecting courses to complete the remaining twenty-four units:

- For a theoretical emphasis, Math. 280A-B-C (Probability Theory) is required.
- For an applied orientation, Math. 270A-B-C (Numerical Mathematics) is recommended.
- A maximum of eight units of approved upper-division applied mathematics courses (see faculty adviser) and Math. 500 (Apprentice Teaching).

Upon the approval of the faculty adviser, all twenty-four units can be graduate-level courses in other departments.

**COMPREHENSIVE EXAMINATIONS**

Two written comprehensive examinations must be passed at the master's level in related course work (approved by a faculty adviser). Instructors of the relevant courses should be consulted for exam dates as they vary on a yearly basis.

**FOREIGN LANGUAGE REQUIREMENT**

There is no foreign language requirement for the M.S. in statistics.

**TIME LIMITS**

Full-time M.S. students are permitted seven quarters in which to complete all requirements. While there are no written time limits for part-time students, the department has the right to intervene and set individual deadlines if it becomes necessary.

**Ph.D. in Mathematics**

**WRITTEN QUALIFYING EXAMINATIONS**

The department offers written qualifying examinations in seven subjects. These are grouped into three areas as follows:

- **Area #1**
  - Complex Analysis (Math. 220A-B-C)
  - Real Analysis (Math. 240A-B-C)

- **Area #2**
  - Algebra (Math. 200A-B-C)
  - Applied Algebra (Math. 202A-B-C)
  - Topology (Math. 290A-B-C)

- **Area #3**
  - Numerical Analysis (Math. 270A-B-C)
  - Statistics (Math. 281A-B-C)

i) Three qualifying exams must be passed. At least one must be passed at the Ph.D. level, and a second must be passed at either the Ph.D. or Provisional Ph.D. level. The third exam must be passed at least at the master's level.

ii) Of the three qualifying exams, there must be at least one from each of Areas #1 and #2. Algebra and Applied Algebra do not count as distinct exams in Area #2.

iii) Students must pass a least two exams from distinct areas with a minimum grade of Provisional Ph.D. (For example, a Ph.D. pass in Real Analysis, Provisional Ph.D. pass in Complex Analysis, M.A. pass in Algebra would NOT satisfy this requirement, but a Ph.D. pass in Real Analysis, M.A. pass in Complex Analysis, Provisional Ph.D. pass in Algebra would, as would a Ph.D. pass in Numerical Analysis, Provisional Ph.D. pass in Applied Algebra, and M.A. pass in Real Analysis.)

iv) All exams must be passed by the September exam session prior to the beginning of the third year of graduate studies. (Thus, there would be no limit on the number of attempts, encouraging new students to take exams when they arrive, without penalty.)

Department policy stipulates that a least one of the exams must be completed with a Provisional Ph.D. pass or better by September following the end of the first year. Anyone unable to comply with this schedule will be terminated from the doctoral program and transferred to one of our Master's programs.

Any Master's student can submit for consideration a written request to transfer into the Ph.D. program when the qualifying exam requirements for the Ph.D. program have been met and a dissertation adviser is found. Approval by the Qualifying Exam and Appeals Committee (QEAC) is not automatic, however.

Exams are typically offered twice a year, one scheduled late in the spring quarter and again in early September (prior to the start of fall quarter). Copies of past exams are made available for purchase in the Graduate Office.

In choosing a program with an eye to future employment, students should seek the assistance of a faculty adviser and take a broad selection of courses including applied mathematics, such as those in Area #3.

**FOREIGN LANGUAGE REQUIREMENT**

A reading knowledge of one foreign language (French, German, or Russian) is required prior to advancing to candidacy. In exceptional cases other languages may be substituted. Testing is administered within the department by faculty who select published mathematical material in one of these languages for a student to translate.

**ADVANCEMENT TO CANDIDACY**

It is expected that by the end of the third year (nine quarters), students should have a field of
research chosen and a faculty member willing to direct and guide them. A student will advance to candidacy after successfully passing the oral qualifying examination, which deals primarily with the area of research proposed but may include the project itself. This examination is conducted by the student’s appointed doctoral committee. Based on their recommendation, a student advances to candidacy and is awarded the C.Phil. degree.

**DISSERTATION AND FINAL DEFENSE**

Submission of a written dissertation and a final examination in which the thesis is publicly defended are the last steps before the Ph.D. degree is awarded. When the dissertation is substantially completed, copies must be provided to all committee members at least four weeks in advance of the proposed defense date. Two weeks before the scheduled final defense, a copy of the dissertation must be made available in the department for public inspection.

**TIME LIMITS**

The normative time for the Ph.D. in mathematics is five years. Students must be advanced to candidacy by the end of eleven quarters. Total university support cannot exceed six years. Total registered time at UCSD cannot exceed seven years.

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**COURSES**

*For course descriptions not found in the 2006–2007 General Catalog, please contact the department for more information.*

All prerequisites listed below may be replaced by an equivalent or higher-level course. The listings of quarters in which courses will be offered are only tentative. Please consult the Department of Mathematics to determine the actual course offerings each year.

**LOWER-DIVISION**

**3C. Pre-Calculus (4)**
Functions and their graphs. Linear and polynomial functions, zeroes, inverse functions, exponential and logarithmic, trigonometric functions and their inverses. Emphasis on understanding algebraic, numerical and graphical approaches making use of graphing calculators. (No credit given if taken after Math. 4C, 1A/10A, or 2A/20A.) Prerequisite: two or more years of high school mathematics or equivalent.

**4C. Pre-Calculus for Science and Engineering (4)**
Review of polynomials. Graphing functions and relations: graphing rational functions, effects of linear changes of coordinates. Circular functions and right triangle trigonometry. Reinforcement of function concept: exponential, logarithmic, and trigonometric functions. Vectors. Conic sections. Polar coordinates. Three lectures, one recitation. (No credit given if taken after Math. 1A/10A or 2A/20A. Two units of credit given if taken after Math. 3C.) Prerequisite: Math Placement Exam qualifying score, or Math. 3C with a grade of C or better.

**10A. Calculus (4)**
Differentiation and integration of algebraic functions. Fundamental theorem of calculus. Applications. (No credit given if taken after Math. 2A/20A. Formerly numbered Math. 1A.) Prerequisites: Math Placement Exam qualifying score, or AP Calculus AB score of 2, or SAT II Math. 2C score of 650 or higher, or Math. 3C with a grade of C or better, or Math. 4C with a grade of C– or better.

**10B. Calculus (4)**
Further applications of the definite integral. Calculus of trigonometric, logarithmic, and exponential functions. Complex numbers. (No credit given if taken after Math. 2B/20B. Formerly numbered Math. 1B.) Prerequisites: AP Calculus AB score of 3, 4, or 5, or Math. 10A with a grade of C– or better, or Math. 20A with a grade of C– or better.

**10C. Calculus (4)**
Vector geometry, velocity, and acceleration vectors. (No credit given if taken after Math. 2C/20C. Formerly numbered Math. 1C.) Prerequisites: AP Calculus BC score of 3, 4, or 5, or Math. 10B with a grade of C– or better, or Math. 20B with a grade of C– or better.

**11. Elementary Probability and Statistics (4)**

**15A. Discrete Mathematics (4)**
Basic discrete mathematical structures: sets, relations, functions, sequences, equivalence relations, partial orders, number systems. Methods of reasoning and proofs: propositional logic, predicate logic, induction, recursion, pigeonhole principle. Infinite sets and diagonalization. Basic counting techniques; permutations and combinations. Applications will be given to digital logic design, elementary number theory, design of programs, and proofs of program correctness. Equivalent to CSE 20. Credit not offered for both Math. 15A and CSE 20. Prerequisites: CSE 8A or CSE 8B or CSE 11. CSE 8B or CSE 11 may be taken concurrently with Math. 15A/CSE 20.

**15B. Mathematics for Algorithm and Systems (4)**
This course introduces mathematical tools for the qualitative and quantitative analysis of algorithms and computer systems. Topics to be covered include basic enumeration and counting techniques; recurrence relations; graph theory; asymptotic notation; elementary applied discrete combinatorics; equivalence to CSE 21. Prerequisite: Math. 15A or CSE 20 or 160A; CSE 12 is strongly recommended for CSE 21.

**18. Computer Animated Statistics (4)**
Students will acquire the basics of statistical analysis by working with computer-simulated models rather than abstract mathematical language. Topics include hypothesis testing, maximum likelihood estimation, sampling, chi-square tests and construction of confidence intervals. Prerequisite: Math. 1B or 10B or 20B.

**20A. Calculus for Science and Engineering (4)**
Foundations of differential and integral calculus of one variable. Functions, graphs, continuity, limits, derivative, tangent line. Applications with algebraic, exponential, logarithmic, and trigonometric functions. Introduction to the integral. (Two credits given if taken after Math. 1A/10A and no credit given if taken after Math. 1B/10B or Math. 1C/10C. Formerly numbered Math. 2A.) Prerequisite: Math Placement Exam qualifying score, or AP Calculus AB score of 2 or 3, or SAT II Math. 2C score of 650 or higher, or Math. 4C with a grade of C– or better, or Math. 10A with a grade of C– or better.

**20B. Calculus for Science and Engineering (4)**
Integral calculus of one variable and its applications, with exponential, logarithmic, hyperbolic, and trigonometric functions. Methods of integration. Infinite series. Polar coordinates in the plane and complex exponentials. (Two units of credits given if taken after Math. 1B/10B or Math. 1C/10C.) Prerequisite: AP Calculus AB score of 3, 4, or 5, or AP Calculus BC score of 3, or Math. 20A with a grade of C– or better, or Math. 10B with a grade of C– or better, or Math. 10C with a grade of C– or better.

**20C. Calculus and Analytic Geometry for Science and Engineering (4)**
Vector geometry, vector functions and their derivatives. Partial differentiation. Maxima and minima. Double integration, Two units of credit given if taken after Math. 10C. Formerly numbered Math. 21C. Prerequisite: AP Calculus BC score of 3, 4, or 5, or Math. 20B with a grade of C– or better.

**20D. Introduction to Differential Equations (4)**

**20E. Vector Calculus (4)**

**20F. Linear Algebra (4)**
Matrix algebra, solution of systems of linear equations by Gaussian elimination, determinants. Linear and affine subspaces; bases of Euclidean spaces. Eigenvalues and eigenvectors, quadratic forms, orthogonal matrices, diagonalization of symmetric matrices. Applications. Computing symbolic and graphical solutions using Matlab. (Zero units given if Math. 2EA previously. Formerly numbered 2EA.) Prerequisite: Math. 20C (or Math. 21C) with a grade of C– or better.
87. Freshman Seminar (1)
The Freshman Seminar Program is designed to provide new students with the opportunity to explore an intellectual topic with a faculty member in a small seminar setting. Freshman seminars are offered in all campus departments and undergraduate colleges, and topics vary from quarter to quarter. Enrollment is limited to 15 to 20 students, with preference given to entering freshman. Prerequisite: none.

UPPER-DIVISION

100A-B. Modern Algebra (4-4-4)
An introduction to the methods and basic structures of higher algebra: sets and mappings, the integers, rational, real and complex numbers, groups, rings (especially polynomial rings) and ideals, fields, real and complex vector spaces, linear transformations, inner product spaces, matrices, triangular form, diagonalization. Both 100 and 103 cannot be taken for credit. Three lectures; one recitation. Prerequisites: Math. 20F and Math. 109 or consent of instructor. (F, W, S)

102. Applied Linear Algebra (4)

103A-B. Modern Applied Algebra (4-4)
Abstract algebra with applications to computation. Set algebra and graph theory. Finite state machines. Boolean algebras and switching theory. Lattices. Groups, rings and fields: applications to coding theory. Recurrent sequences. Three lectures; one recitation. Both 100 and 103 cannot be taken for credit. Prerequisites: Math. 20F and Math. 109 (may be taken concurrently). (F, W)

104A-B-C. Number Theory (4-4-4)
Topics from number theory with applications and computing. Possible topics are: congruences, reciprocity laws, quadratic forms, prime number theorem, Riemann zeta function, Fermat's conjecture, diophantine equations, Gaussian sums, algebraic integers, unique factorization into prime ideals in algebraic number fields, class number, units, splitting of prime ideals in extensions, quadratic and cyclotomic fields, partitions. Possible applications are Fast Fourier Transform, signal processing, coding, cryptography. Three lectures. Prerequisite: consent of instructor.

107A-B. Computer Algebra (4)
An introduction to algebraic computation. Computational aspects of groups, rings, fields, etc. Data representation and algorithms for symbolic computation. Polynomials and their arithmetic. The use of a computer algebra system as an experimental tool in mathematics. Programming using algebra systems. Prerequisite: prior or concurrent enrollment in the Math. 100 or 103 sequence.

109. Mathematical Reasoning (4)
This course uses a variety of topics in mathematics to introduce the students to rigorous mathematical proof, emphasizing quantifiers, induction, negation, proof by contradiction, naive set theory, equivalence relations and epsilon-delta proofs. Required of all departmental majors. Prerequisite: Math. 20F.

110. Introduction to Partial Differential Equations (4)
Fourier series, orthogonal expansions, and eigenvalue problems. Sturm-Liouville theory. Separation of variables for partial differential equations of mathematical physics, including topics on Bessel functions and Legendre polynomials. Prerequisites: Math. 20D (or 21D) and 20F, or consent of instructor. (F, S)

111A-B. Mathematical Modeling (4-4)
An introduction to mathematical modeling in the physical and social sciences, concentrating on one or more topics that vary from year to year. Students work on independent or group projects. May be repeated for credit when topics change. Prerequisites: Math. 20D and Math. 20F or consent of instructor.

120A. Elements of Complex Analysis (4)
Complex numbers and functions. Analytic functions, harmonic functions, elementary conformal mappings. Complex integration. Power series. Cauchy's theorem. Cauchy's formula. Residue theorem. Three lectures; one recitation. Prerequisite or co-registration: Math. 20F or consent of instructor. (F, W)

120B. Applied Complex Analysis (4)
Applications of the Residue Theorem. Conformal mapping and applications to potential theory, flows, and temperature distributions. Fourier transformations. Laplace transformations, and applications to integral and differential equations. Selected topics such as Poisson's formula. Dirichlet problem. Neumann's problem, or special functions. Three lectures; one recitation. Prerequisite: Math. 120A. (W, S)

130A. Ordinary Differential Equations (4)
Linear and nonlinear systems of differential equations. Stability theory, perturbation theory. Applications and introduction to numerical solutions. Three lectures. Prerequisites: Math. 20D/21D and 20F. (F)

130B. Ordinary Differential Equations (4)
Existence and uniqueness of solutions to differential equations. Local and global theorems of continuity and differentiability. Three lectures. Prerequisites: Math. 20D/21D and 20F, and Math. 130A. (W)

131. Variational Methods in Optimization (4)
Maximum-minimum problems. Normalized vector spaces, functionals, Gateaux variations. Euler-Lagrange multiplier theorem for an extremum with constraints. Calculus of variations via the multiplier theorem. Applications may be taken from a variety of areas such as the following: applied mechanics, elasticity, economics, production planning and resource allocation, aeronautics, rocket control, physics, Fermat's principle and Hamilton's principle, geometry, geodesic curves, control theory, elementary bang-bang problems. Three lectures; one recitation. Prerequisites: Math. 20D/21D and 20F or consent of instructor. (S)

132A. Elements of Partial Differential Equations and Integral Equations (4)
Basic concepts and classification of partial differential equations. First order equations, characteristics. Hamilton-Jacobi theory, Laplace's equation, wave equation, heat equation. Separation of variables, eigenfunction expansions, existence and uniqueness of solutions. Three lectures; Prerequisite: Math. 110 or consent of instructor. (W)

132B. Elements of Partial Differential Equations and Integral Equations (4)
Relation between differential and integral equations, some classical integral equations, Volterra integral equations, partial equations of the second kind, degenerate kernels, Fredholm alternative, Neumann-Liouville series, the resolvent kernel. Three lectures. Prerequisite: Math. 132A. (S)

140A-B-C. Foundations of Analysis (4-4-4)
Axioms, the real number system, topology of the real line, metric spaces, continuous functions, sequences of functions, differentiation, Riemann-Stieltjes integration, partial differentiation, multiple integration, Jacobians. Additional topics at the discretion of the instructor: power series, Fourier series, successive approximations of other infinite processes. Three lectures; one recitation. Prerequisites: Math. 20F and Math. 109 or consent of instructor. Credit cannot be obtained for both Math. 140A-B and 142A-B. (F, W, S)

142A-B. Advanced Calculus (4-4)

150A. Differential Geometry (4)
Differential geometry of curves and surfaces. Gauss and mean curvatures, geodesics, parallel displacement, Gauss-Bonnet theorem. Three lectures. Prerequisites: Math. 20E with a grade of C- or better and Math. 20F with a grade of C- or better, or consent of instructor. (F)

150B. Calculus on Manifolds (4)
Calculating functions of several variables, inverse function theorem. Further topics, selected by the instructor, such as exterior differential forms, Stokes' theorem, manifolds, Sard's theorem, elements of differential topology, singularities of maps, catastrophes, further topics in differential geometry, topics in geometry of physics. Three lectures. Prerequisite: Math. 150A. (W)

151. Topics in Geometry (4)
A topic, selected by the instructor, from Euclidean geometry, non-Euclidean geometry, projective geometry, algebraic geometry, or other geometries. May be repeated for credit with a different topic. Three lectures. Prerequisite: consent of instructor. (S)

152. Applicable Mathematics and Computing (4)
This course will give students experience in applying theory to real world applications such as Internet and wireless communication problems. The course will incorporate talks by experts from industry and students will be helped to carry out independent projects. Topics include graph visualization, labelling, and embeddings, random graphs and randomized algorithms. May be taken 3 times for credit. Prerequisites: Math. 20D or 21D, and 20F or consent of instructor.

153. Geometry for Secondary Teachers (4)
Two- and three-dimensional Euclidean geometry is developed from one set of axioms. Pedagogical issues will emerge from the mathematics and be addressed using current research in teaching and learning geometry. This course is designed for prospective secondary school mathematics teachers. Prerequisite: Math. 109.

154. Discrete Mathematics and Graph Theory (4)
Basic concepts in graph theory. Combinatorial tools, structures in graphs (Hamiltonian cycles, perfect matching). Properties of graphs and applications in basic algorithmic problems (planarity, k-colorability,
traveling salesman problem). Prerequisites: Math. 20F and Math. 109, or consent of instructor.

155A. Computer Graphics (4) Beziers and control lines, de Casteljau construction for subdivision, elevation of degree, control points of Hermite curves, barycentric coordinates, rational curves. Three lectures, one recitation, and approximately eight laboratory hours per week. Prerequisites: Math. 20F and programming experience. [Warning: There are duplicate credit restrictions on this course. See section on Duplication of Credit.] (F)

155B. Topics in Computer Graphics (4) Spline curves, spline interpolation, affine and affine cross ratios, polar forms (blossoming), the Oslo algorithm for knot insertion, NURBS and geometric continuity. Three lectures, one recitation, and approximately eight laboratory hours per week. Prerequisite: Math. 20A or consent of instructor. (W)


161. Mathematical Computing (2 or 4) Programming in higher level mathematical language such as Mathematical: Lists, Functions, Expressions, Recursion, Iteration, graphics, packages. Application to diverse areas of mathematics such as differential equations, dynamical systems, fractals, chaos, probability, financial models. Prerequisite: Math. 20A-B, 21C-D, 20E-F or equivalent.

163. History of Mathematics (4) Topics will vary from year to year in areas of mathematics and their development. Topics may include the evolution of mathematics from the Babylonian period to the eighteenth century using original sources, a history of the foundations of mathematics and the development of modern mathematics. Prerequisite: Math. 20B or consent of instructor. (S)

165. Introduction to Set Theory (4) Sets, relations, and functions. Partial, linear, and well-orders. The axiom of choice, proof by induction and definition by recursion. Cardinal and ordinal numbers and their arithmetic. Prerequisite: Math. 100A or 140A or 103, or consent of instructor.

166. Intro to the Theory of Computation (4) Introduction to formal languages; regular languages; regular expressions, finite automata, minimization, closure properties, decision algorithms, and non-regular languages; context-free languages, context-free grammars, push-down automata, parsing theory, closure properties, and noncontext-free languages; computable languages; parsing machines, recursive functions, Church's thesis, undecidability and the halting problem. Equivalent to CSE 105. Prerequisites: CSE 8B or 9B or 10 or 65 or 62B AND CSE 20 or 160A or Math. 15A or 109 or 100A or 103A.

168A. Topics in Applied Mathematics-Computer Science (4) Topics to be chosen in areas of applied mathematics and mathematical aspects of computer science. May be repeated once for credit with different topics. Three lectures, one recitation. Prerequisite: consent of instructor. (W,S)

170A. Numerical Linear Algebra (4) Analysis of numerical methods for linear algebraic systems and least squares problems. Orthogonalization methods. Ill-conditioned problems. Eigenvalue and singular value computations. Three lectures, one recitation. Prerequisites: Math. 20F and knowledge of programming. (F,S)

170B. Numerical Analysis (4) Rounding and discretization errors. Calculation of roots of polynomials and nonlinear equations. Interpolation. Approximation of functions. Three lectures, one recitation. Prerequisites: Math. 20F and knowledge of programming. (W)


171A-B. Mathematical Programming–Numerical Optimization (4-4) Mathematical optimization and applications. Linear programming, the simplex method, duality. Nonlinear programming, Kuhn-Tucker theorem. Selected topics from integer programming, network flows, transportation problems, inventory problems, and other applications. Three lectures, one recitation. Prerequisites: Math. 20F and knowledge of programming.


173. Mathematical Software–Scientific Programming (4) Development of high quality mathematical software for the computer solution of mathematical problems. Three lectures, one recitation. Prerequisites: Math. 170A or Math. 174 and knowledge of FORTRAN. (W)

174. Numerical Methods in Science and Engineering (4) Floating point arithmetic, linear equations, interpolation, integration, differential equations, nonlinear equations, optimization, least squares. Students may not receive credit for both Math. 174 and Physics 105 or MAE 153 or 154. Students may not receive credit for Math. 174 if Math. 170 A,B, or C has already been taken. Prerequisites: Math. 20D (21D) and Math. 20F.

176. Advanced Data Structures (4) Descriptive and analytical presentation of data structures and algorithms. Lists, tables, priority queues, disjunct subsets, and dictionaries data types. Data structuring techniques include linked lists, arrays, hashing, and trees. Performance evaluation involving worst case, average and expected case, and amortized analysis. Credit not offered for both Math. 176 and CSE 100. Equivalent to CSE 100. Prerequisites: CSE 12, CSE 21, or Math. 15B, and CSE 30, or consent of instructor.

180A. Introduction to Probability (4) Probability spaces, random variables, independence, conditional probability, distribution, expectation, joint distributions, central-limit theorem. Three lectures. Prerequisites: Math. 20D/21D. (Warning: There are duplicate credit restrictions on this course. See section on Duplication of Credit.) (F)

180B. Introduction to Probability (4) Random vectors, multivariate densities, covariance matrix, multivariate normal distribution. Random walk, Poisson process. Other topics if time permits. Three lectures. Prerequisites: Math. 180A and Math. 20E. (W)

180C. Introduction to Probability (4) Markov chains in discrete and continuous time, random walk, recurrent events. If time permits, topics chosen from stationary normal processes, branching processes, queuing theory. Three lectures. Prerequisite: Math. 180B. (S)

181A. Introduction to Mathematical Statistics (4) Random samples, linear regression, least squares, testing hypotheses, and estimation. Neyman-Pearson lemma, likelihood ratios. Three lectures, one recitation. Prerequisites: Math. 180A and 20F. [Warning: There are duplicate credit restrictions on this course. See section on Duplication of Credit.] (W)


181C. Mathematical Statistics (4) Nonparametric Statistics. Topics covered may include the following: Classical rank test, rank correlations, permutation tests, distribution free testing, efficiency, confidence intervals, nonparametric regression and density estimation, resampling techniques (bootstrap, jackknife, etc.) and cross validations. Prerequisites: Math. 181A, 181B previously or concurrently.


183. Statistical Methods (4) Introduction to probability. Discrete and continuous random variables–binomial, Poisson and Gaussian distributions. Central limit theorem. Data analysis and inferential statistics: graphical techniques, confidence intervals, hypothesis tests, curve fitting. (Credit not offered for Math. 183 if Econ. 120A, Math. 180A, or Math. 181A previously or concurrently.) Prerequisite: Math. 20C (21C). (F,S)


186. Probability Statistics for Bioinformatics (4) This course will cover an introduction to probability and statistics, the use of discrete and random variables, different types of distributions, data analysis and inferential statistics, likelihood estimators and scoring matrices with applications to biological problems. Introduction to probability, Binomial, Poisson, and Gaussian distributions, central limit theorem, applications to sequence and functional analysis of genomes and genetic epidemiology. Prerequisite: Math. 20A, Math. 20B, Math. 20C (21C).


188. Design and Analysis of Algorithms (4)
Introduction to the design and analysis of efficient algorithms. Basic techniques for analyzing the time requirements of algorithms. Algorithms for sorting, searching, and pattern matching, algorithms for graphs and networks. NP-complete problems. Equivalent to CSE 101. Prerequisites: CSE 100 or Math. 176A for Math. 188; CSE 21, CSE 21E, and CSE 100 for CSE 101.

190. Introduction to Topology (4)
Topological spaces, subspaces, products, sums and quotient spaces. Compactness, connectedness, separation axioms. Selected further topics such as fundamental group, classification of surfaces, Morse theory, topological groups. May be repeated for credit once when topics vary, with consent of instructor. Three lectures. Prerequisite: Math. 109 or consent of instructor. (W)

191. Topics in Topology (4)
Topics to be chosen by the instructor from the fields of differential algebraic, geometric, and general topology. Three lectures. Prerequisite: Math. 190 or consent of instructor. (S)

193A. Actuarial Mathematics (4)
Probabilistic Foundations of Insurance. Short-term risk models. Survival distributions and life tables. Introduction to life insurance. Prerequisite: Math. 180A or 183, or consent of instructor.

193B. Actuarial Mathematics (4)
Life Insurance and Annuities: Analysis of premiums and premium reserves. Introduction to multiple life functions and decrement models as time permits. Prerequisite: Math. 193A.

194. The Mathematics of Finance (4)
Introduction to the mathematics of financial models. Basic probabilistic models and associated mathematical machinery will be discussed, with emphasis on discrete time models. Concepts covered will include conditional expectation, martingales, optimal stopping, arbitrage pricing, hedging, European and American options. Prerequisites: Math. 150D (21D), Math. 206, and Math. 180A or 183.

195. Introduction to Teaching in Mathematics (4)
Students will be responsible for and teach a class section of a lower-division mathematics course. They will also attend a weekly meeting on teaching methods. (Does not count towards a minor or major) Five lectures, one recitation. Prerequisite: consent of instructor. (F,W,S)

196. Student Colloquium (1)
A variety of topics and current research results in mathematics will be presented by guest lecturers and students under faculty direction. Prerequisite: upper-division status.

197. Mathematics Internship (2 or 4)
An enrichment program which provides work experience with public/private sector employers. Subject to the availability of positions, students will work in a local company under the supervision of a faculty member and site supervisor. Units may not be applied towards major graduation requirements. Prerequisites: completion of 90 units, 2 upper-division mathematics courses, an overall 2.5 UCSD G.P.A., consent of mathematics faculty coordinator, and submission of written contract. Department stamp required.

199. Independent Study for Undergraduates (2 or 4)
Independent reading in advanced mathematics by individual students. Three periods. (P/NP grades only.) Prerequisite: permission of department. (F,W,S)

199H. Honors Thesis Research for Undergraduates (2-4)
Honors thesis research for seniors participating in the Honors Program. Research is conducted under the supervision of a mathematics faculty member. Prerequisites: admission to the Honors Program in mathematics, department stamp.

GRADUATE

200A-B-C. Algebra (4-4-4)
Group actions, factor groups, polynomial rings, linear algebra, rational and Jordan canonical forms, unitary and Hermitian matrices, Sylvow theorems, finitely generated abelian groups, unique factorization, Galois theory, solvability by radicals, Hilbert Basis Theorem, Hilbert Nullstellensatz, Jacobson radical, semisimple Artinian rings. Prerequisite: consent of instructor.

201A. Basic Topics in Algebra (4)
Recommended for all students specializing in algebra. Basic topics include categorical algebra, commutative algebra, group representations, homological algebra, nonassociative algebra, ring theory. Prerequisites: Math. 200A/B-C or consent of instructor. (F,W,S)

202B-C. Applied Algebra (4-4)
Algebra from a computational perspective using Maple, Mathematica and Matlab. Groups, rings, linear algebra, rational and Jordan forms, unitary and Hermitian matrices, matrix decompositions, perturbation of eigenvalues, group representations, symmetric functions, fast Fourier transform, commutative algebra, Grobner basis, finite fields. Prerequisite: consent of instructor.

203A-B-C. Algebraic Geometry (4-4-4)
Places, Hilbert Nullstellensatz, varieties, product of varieties: correspondences, normal varieties. Divisors and linear systems; Riemann-Roch theorem; resolution of singularities of curves. Grothendieck schemes; cohomology, Hilbert schemes; Picard schemes. Prerequisites: Math. 200A-B-C. (F,W,S)

204. Topics in Number Theory (4)
Topics in analytic number theory, such as zeta functions and L-functions and the distribution of prime numbers, zeros of zeta functions and Siegel's theorem, transcendence theory, modular forms, finite and infinite symmetric spaces. Prerequisite: consent of instructor.

205. Topics in Algebraic Number Theory (4)
Topics in algebraic number theory, such as cyclotomic and Kummer extensions, class number, units, splitting of primes in extensions, zeta functions of number fields and the Brauer-Siegel Theorem, class field theory, elliptic curves and curves of higher genus, complex multiplication. Prerequisite: consent of instructor.

207A-B-C. Topics in Algebra (4-4-4)
In recent years, topics have included number theory, commutative algebra, noncommutative rings, homological algebra, and Lie groups. May be repeated for credit with consent of adviser. Prerequisite: consent of instructor.

208. Seminar in Algebra (1-4)
Prerequisite: consent of instructor. (S/U grades permitted.)

209. Seminar in Number Theory (1 to 4)
Prerequisite: consent of instructor. (S/U grades permitted.)

210A. Mathematical Methods in Physics and Engineering (4)
Complex variables with applications. Analytic functions, Cauchy's theorem, Taylor and Laurent series, residue theorem and contour integration techniques, analytic continuation, argument principle, conformal mapping, potential theory, asymptotic expansions, method of steepest descent. Prerequisites: Math. 200EF, 140A/142A or consent of instructor.

210B. Mathematical Methods in Physics and Engineering (4)
Linear algebra and functional analysis. Vector spaces, orthonormal bases, linear operators and matrices, eigenvalues and diagonalization, least squares approximation, infinite-dimensional spaces, completeness, integral equations, spectral theory, Green's functions, distributions, Fourier transform. Prerequisite: Math. 210A or consent of instructor. (W)

210C. Mathematical Methods in Physics and Engineering (4)
Calculus of variations: Euler-Lagrange equations, Noether's theorem. Fourier analysis of functions and distributions in several variables. Partial differential equations: Laplace, wave, and heat equations; fundamental solutions (Green's functions); well-posed problems. Prerequisite: Math. 210B or consent of instructor. (S)

211. Fourier Analysis on Finite Groups (4)
Applied group representations. Emphasis on the integers, mod n, finite matrix groups. Applications may include: the fast Fourier transform, digital signal processing, finite difference equations, spectral graph theory, error-correcting codes, vibrating systems, finite wavelet transforms. Prerequisite: none.

212A. Introduction to the Mathematics of Systems and Control (4)
Linear and nonlinear systems, and their input-output behavior, linear continuous time and discrete-time systems, reachability and controllability for linear systems, feedback and stabilization, eigenvalue placement, nonlinear controllability, feedback linearization, disturbance rejection, nonlinear stabilization, Lyapunov functions, linearization principle for stability. Prerequisites: Math. 102 or equivalent, Math. 120A or equivalent, Math. 142A or equivalent.

212B. Introduction to the Mathematics of Systems and Control (4)

212C. Introduction to the Mathematics of Systems and Control (4)
Topics of current interest on systems theory, control, and estimation to be chosen by instructor. Prerequisite: Math. 212B.
216. Topics in Pure Mathematics (4) This course brings together graduate students, postdocs, and faculty to examine a current research topic of broad interest. Previously covered topics include: noncommutative geometry, loop groups, geometric quantization. Prerequisite: consent of instructor.

217A. Topics in Applied Mathematics (4) In recent years, topics have included applied complex analysis, special functions, and asymptotic methods. May be repeated for credit with consent of adviser. Prerequisite: consent of instructor.

220A-B-C. Complex Analysis (4-4-4) Complex numbers and functions. Cauchy theorem and its applications, calculus of residues, expansions of analytic functions, analytic continuation, conformal mapping and Riemann mapping theorem, harmonic functions. Dirichlet principle, Riemann surfaces. Prerequisites: Math. 140A-B or consent of instructor. (F,WS)

221A-B. Topics in Several Complex Variables (4-4-4) Formal and convergent power series, Weierstrass preparation theorem; Cartan-Ruckert theorem, analytic sets; mapping theorems; domains of holomorphy; proper holomorphic mappings; complex manifolds; modifications. Prerequisites: Math. 200A and 220A-B-C or consent of instructor.

229. Computing Technology for Mathematics (2) Preparation for making effective use of computer technology in research and teaching of mathematics. UNIX basics, document preparation using TEX, Internet resources, HTML, computer technology in teaching. Prerequisite: graduate status in mathematics.

231A-B-C. Partial Differential Equations (4-4-4) Existence and uniqueness theorems. Cauchy-Kowalewski theorem, first order systems. Hamilton-Jacobi theory, initial value problems for hyperbolic and parabolic systems, boundary value problems for elliptic systems. Green’s function, eigenvalue problems, perturbation theory. Prerequisites: Math. 210A-B or 240A-B-C or consent of instructor.

233. Singular Perturbation Theory for Differential Equations (4) Multivariable techniques, matching techniques and averaging techniques, including various approaches to proofs of asymptotic correctness, for singular perturbation problems including initial value problems with nonuniformities at infinity, initial value problems with initial nonuniformities, two point boundary value problems, and problems for partial differential equations. Applications taken from celestial mechanics, oscillatory problems, fluid dynamics, elasticity, and applied mechanics. Prerequisites: Math. 130A-B or 132A-B or consent of instructor. (S/U grades permitted.) (S)

237A-B-C. Topics in Differential Equations (4-4-4) May be repeated for credit with consent of adviser. Prerequisite: consent of instructor.

240A-B-C. Real Analysis (4-4-4) Lebesgue integral and Lebesgue measure, Fubini theorems, functions of bounded variation, Stieltjes integral, derivatives and indefinite integrals, the spaces L1 and C, equi-continuous families, continuous linear functionals, general measures and integrations. Prerequisites: Math. 140A-B-C, (F,WS)

241A-B. Functional Analysis (4-4) Metric spaces and contraction mapping theorem; closed graph theorem; uniform boundedness principle; Hahn-Banach theorem; representation of continuous linear functionals; conjugate space, weak topologies; extreme points; Krein-Milman theorem; fixed-point theorems; Riesz convexity theorem; Banach algebras. Prerequisites: Math.240A-B-C or consent of instructor.

247A-B-C. Topics in Real Analysis (4-4) In recent years, topics have included Fourier analysis, distribution theory, martingale theory, operator theory. May be repeated for credit with consent of adviser. Prerequisite: consent of instructor.

248. Seminar in Real Analysis (1 to 4) Prerequisite: consent of instructor. (S/U grades permitted.)

250A-B-C. Differential Geometry (4-4-4) Differential manifolds, Sard theorem, tensor bundles, Lie derivatives, DeRham theorem, connections, geo- desics, Riemannian metrics, curvature tensor and sectional curvature, completeness, characteristic classes. Differential manifolds immersed in Euclidean space. Prerequisite: consent of instructor. (F,WS)

251A-B-C. Lie Groups (4-4-4) Lie groups, Lie algebras, exponential map, subgroup subalgebra correspondence, adjoint group, universal enveloping algebra. Structure theory of semi-simple Lie groups, global decompositions, Weyl group. Geometry and analysis on symmetric spaces. Prerequisites: Math. 200 and 250 or consent of instructor. (F,WS)

256. Seminar in Lie Groups and Lie Algebras (2 to 4) Various topics in Lie groups and Lie algebras, including structure theory, representation theory, and applications. Prerequisite: consent of instructor. (F,WS)

257A. Topics in Differential Geometry (4) In recent years, topics have included Morse theory and general relativity. May be repeated for credit with consent of adviser. Prerequisite: consent of instructor.

257B. Topics in Differential Geometry (4) In recent years, topics have included Morse theory and general relativity. May be repeated for credit with consent of adviser. Math. 257A must be taken before Math. 257B. Prerequisite: consent of instructor.

257C. Topics in Differential Geometry (4) In recent years, topics have included Morse theory and general relativity. May be repeated for credit with consent of adviser. Math. 257B must be taken before Math. 257C. Prerequisite: consent of instructor.

258. Seminar in Differential Geometry (1 to 4) Various topics in differential geometry. Prerequisite: consent of instructor.

259A-B-C. Geometrical Physics (4-4-4) Manifolds, differential forms, homology, deRham’s theorem. Riemannian geometry, harmonic forms. Lie groups and algebras, connections in bundles, homotopy sequence of a bundle, Chern classes. Applications selected from Hamiltonian and continuum mechanics, electromagnetism, thermodynamics, special and general relativity, Yang-Mills fields. Prerequisite: graduate standing in mathematics, physics, or engineering, or consent of instructor.

261A-B. Combinatorial Algorithms (4-4) Lexicographic order, backtracking, ranking algorithms, isomorphism rejection, sorting, orderly algorithms, network flow and related topics, constructive Polya theory, inclusion-exclusion and sieving methods, Mobius inversion, generating functions, algorithmic graph theory, trees, recursion, depth first search and applications, matroids. Prerequisites: CSE 160A-B or Math. 184A-B or consent of instructor. (F,WS)

262A-B. Topics in Combinatorial Mathematics (4-4-4) Development of a topic in combinatorial mathematics starting from basic principles. Problems of enumeration, existence, construction, and optimization with regard to finite sets. Some familiarity with computer programming desirable but not required. Prerequisites: Math. 100A-B-C.

263. History of Mathematics (4) Mathematics in the nineteenth century from the original sources. Foundations of analysis and commutative algebra. For algebra the authors studied will be Lagrange, Ruffini, Gauss, Abel, Galois, Dirichlet, Kummer, Kronecker, Dedekind, Weber, M. Noether, Hilbert, Steinitz, Artin, E. Noether. For analysis they will be Cauchy, Fourier, Bolzano, Dirichlet, Riemann, Weierstrass, Heine, Cantor, Peano, Hilbert. Prerequisites: Math. 100A-B, Math. 140A-B-S.

264A-B-C. Combinatorics (4-4-4) Topics from partially ordered sets, Mobius functions, simplicial complexes and shell ability. Enumeration, formal power series and formal languages, generating functions, partitions. Lagrange inversion, exponential structures, combinatorial species. Finite operator methods, q-analogues, Polya theory, Ramsey theory. Representation theory of the symmetric group, symmetric functions and operations with Schur functions. (F,WS)

267A-B-C. Topics in Mathematical Logic (4-4-4) Topics chosen from recursion theory, model theory, and set theory. May be repeated with consent of adviser. Prerequisite: consent of instructor. (S/U grades permitted.)

268. Seminar in Logic (1 to 4) Prerequisite: consent of instructor. (S/U grades permitted.)

269. Seminar in Combinatorics (1 to 4) Prerequisite: consent of instructor. (S/U grades permitted.)

270A-B-C. Numerical Mathematics (4-4-4) Error analysis of the numerical solution of linear equations and least squares problems for the full rank and rank deficient cases. Error analysis of numerical methods for eigenvalue problems and singular value problems. Error analysis of numerical quadrature and of the numerical solution of ordinary differential equations. Prerequisites: Math. 20F and knowledge of programming.

271A-B-C. Numerical Optimization (4-4-4) Formulation and analysis of algorithms for constrained optimization. Optimality conditions; linear and quadratic programming; interior methods; penalty and barrier function methods; sequential quadratic programming methods. Prerequisite: consent of instructor. (F,WS)

272A-B-C. Numerical Partial Differential Equations (4-4-4) The numerical solution of elliptic, parabolic, and hyperbolic partial differential equations; discretization and solution techniques. Prerequisite: consent of instructor. (F,WS)

273A-B-C. Scientific Computation (4-4-4) Continuum mechanics models of physical and biological systems, finite element methods and approximation theory, complexity of iterative methods for linear and nonlinear equations, continuation methods, adaptive methods, parallel computing, and scientific
277A. Topics in Numerical Mathematics (4)
Topics vary from year to year. May be repeated for credit with consent of adviser. Prerequisite: consent of instructor.

278. Seminar in Numerical Mathematics (1 to 4)
Prerequisite: consent of instructor. (S/U grades permitted.)

280A-B-C. Probability Theory (4-4-4)
Probability measures; Borel fields; conditional probabilities; sums of independent random variables; limit theorems; zero-one laws; stochastic processes. Prerequisites: advanced calculus and consent of instructor. (F,W,S)

281A. Mathematical Statistics (4)
Statistical models, sufficiency, efficiency, optimal estimation, least squares and maximum likelihood, large sample theory. Prerequisites: advanced calculus and basic probability theory or consent of instructor.

281B. Mathematical Statistics (4)
Hypothesis testing and confidence intervals, one-sample and two-sample problems. Bayes theory, statistical decision theory, linear models and regression. Prerequisites: advanced calculus and basic probability theory or consent of instructor.

281C. Mathematical Statistics (4)
Nonparametrics: tests, regression, density estimation, bootstrap and jackknife. Introduction to statistical computing using S plus. Prerequisites: advanced calculus and basic probability theory or consent of instructor.

282A-B. Applied Statistics (4-4)
Sequence in applied statistics. First quarter: general theory of linear models with applications to regression analysis. Second quarter: analysis of variance and covariance and experimental design. Third quarter: further topics to be selected by instructor. Emphasis throughout is on the analysis of actual data. Prerequisite: Math. 181B or equivalent or consent of instructor. (S/U grades permitted.)

283. Statistical Methods in Bioinformatics (4)
This course will cover material related to the analysis of modern genomic data; sequence analysis, gene expression/functional genomics analysis, and gene mapping/applied population genetics. The course will focus on statistical modeling and inference issues and not on database mining techniques. Prerequisites: one year of calculus, one statistics course or consent of instructor.

285A-B. Stochastic Processes (4-4)
Elements of stochastic processes, Markov chains, hidden Markov models, Poisson point processes, renewal processes martingales, Brownian motion, Gaussian processes, Kalman filter. Other topics to be selected by instructor depending on interest of class. Prerequisites: Math. 180A (or equivalent basic probability course) or consent of instructor.

286. Stochastic Differential Equations (4)

287A. Time Series Analysis (4)
Discussion of finite parameter schemes in the Gaussian and non-Gaussian context. Estimation for finite parameter schemes. Stationary processes and their spectral representation. Spectral estimation. Prerequisite: Math. 181B or equivalent or consent of instructor.

287B. Multivariate Analysis (4)

288. Seminar in Probability and Statistics (1 to 4)
Prerequisite: consent of instructor. (S/U grades permitted.)

289A-B. Topics in Probability and Statistics (4-4)
In recent years, topics have included Markov processes, martingale theory, stochastic processes, stationary and Gaussian processes, ergodic theory. May be repeated for credit with consent of adviser.

290A-B-C. Topology (4-4-4)
Point set topology, including separation axioms, compactness, connectedness. Algebraic topology, including the fundamental group, covering spaces, homology and cohomology. Homotopy or applications to manifolds as time permits. Prerequisites: Math. 100A-B-C and Math. 140A-B-C. (F,W,S)

291A-B-C. Topics in Topology (4)
In recent years, topics have included generalized cohomology theory, spectral sequences, K-theory, homotopy theory. Prerequisites: consent of instructor.

292. Seminar in Topology (1 to 2)
A variety of topics and current research results in topology. Prerequisites: consent of instructor.

294. The Mathematics of Finance (4)
Introduction to the mathematics of financial models. Hedging, pricing by arbitrage. Discrete and continuous stochastic models. Martingales, Brownian motion, stochastic calculus. Black-Scholes model, adaptations to dividend paying equities, currencies and coupon-paying bonds, interest rate market, foreign exchange models. Prerequisite: Math. 180A (or equivalent probability course) or consent of instructor.

295. Special Topics in Mathematics (1 to 4)
A variety of topics and current research results in mathematics will be presented by staff members and students under faculty direction.

296. Student Colloquium (1 to 2)
A variety of topics and current research in mathematics will be presented by guest lecturers and students under faculty direction. Prerequisites: for one unit—upper-division status or consent of instructor (may only be taken P/NP); for two units—consent of instructor, standard grading option allowed.

297. Mathematics Graduate Research Internship (2-4)
An enrichment program which provides work experience with public/private sector employers and researchers. Under supervision of a faculty adviser, students provide mathematical consultation services. Prerequisites: consent of instructor.

299. Reading and Research (1 to 12)
Independent study and research for the doctoral dissertation. One to three credits will be given for independent study (reading) and one to nine for research. Prerequisite: consent of instructor. (S/U grades permitted.)

Teaching of Mathematics

500. Apprentice Teaching (1 to 4)
Supervised teaching as part of the mathematics instructional program on campus (or, in special cases such as the CTF program, off campus). Prerequisite: consent of adviser. (S/U grades only.)